Electrons in ferromagnets with domain walls

2003 J. Phys. A: Math. Gen. 36 9263

(http://iopscience.iop.org/0305-4470/36/35/312)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **36** (2003) 9263–9274 PII: S0305-4470(03)59048-5

Electrons in ferromagnets with domain walls

V K Dugaev[1,2], J Barnaś[3,4] and J Berakdar[1]

[1] Max-Planck-Institut für Mikrostukturphysik, Weinberg 2, 06120 Halle, Germany
[2] Institute for Problems of Materials Science, Vilde 5, 58001 Chernovtsy, Ukraine
[3] Department of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
[4] Institute of Molecular Physics, Polish Academy of Sciences, Smoluchowskiego 17, 60-179 Poznań, Poland

E-mail: vdugaev@mpi-halle.de

Received 31 January 2003, in final form 31 March 2003
Published 20 August 2003
Online at stacks.iop.org/JPhysA/36/9263

Abstract

Domain walls can significantly modify electronic properties of ferromagnetic metals. In this paper we consider theoretically the influence of domain walls on transport properties of ferromagnetic materials and the results are compared with recent experiments. In the case of diffusive transport through a thick domain wall, the semiclassical approximation is applied and a local spin transformation is performed, which replaces the system with a domain wall by the corresponding system without a domain wall but with an additional gauge field. Due to a redistribution of single-particle electron states at the wall, one obtains then either negative or positive contributions to resistivity. The situation is different for very narrow and/or constrained domain walls. In such a case, the semiclassical approximation is not valid. Instead of this the approach based on scattering matrix is applied. The domain wall then gives rise to a large positive contribution to electrical resistivity. The corresponding magnetoresistance can be therefore very large, which is in agreement with recent experiments. The limiting case of narrow domain walls in systems with a single conduction channel is analysed in detail, with the effects due to electron–electron interaction taken into account. In this particular case the magnetoresistance due to a domain wall can be extremely large.

PACS numbers: 76.60.Ch, 75.70.Cn, 75.75.+a

1. Introduction

Domain walls (DWs) can have a significant influence on the behaviour of conducting electrons in ferromagnetic metals. On the other hand, conduction electrons can have an impact on DWs as well. It has been well known for a long time that DWs in a ferromagnetic metal influence its electronic transport properties by producing an additional contribution to electrical resistivity.

Since DWs give rise to electron scattering [1, 2], one could expect that this contribution is positive. This expectation was also supported by early experiments. It was only very recently that a single DW contribution to electrical resistivity could be extracted in a controllable way from the total resistance [3–6]. Surprisingly, it turned out that the resistance of a system with DWs in some cases was smaller than in the absence of DWs [3, 4], whereas in other cases it was larger [7–9]. This intriguing observation led to considerable theoretical interest in electronic transport through DWs [10–14]. The interest is additionally stimulated by possible applications of the associated magnetoresistance in magnetoelectronic devices. This is because creation and destruction of DWs can be controlled by a weak magnetic field. The corresponding magnetoresistance can then be either positive or negative.

Recent experiments on magnetic point contacts showed that constrained DWs formed at the very contact between ferromagnetic wires give an unexpectedly large contribution to electrical resistivity, and consequently lead to large negative magnetoresistance [15]. The characteristic feature of DWs in point contact geometry is their very small width (a few Ångstroms) [16, 17], which is much smaller than the DWs width in bulk materials, thin films or in wires.

In the following we will describe theoretically basic features of the electronic transport through DWs, and will present an explanation of the above described experimental observations. Two limits will be analysed in detail—the limit of thick DW, when electronic transport through the wall is diffusive, and the limit of narrow DW, when the transport is ballistic. In the former case the theoretical treatment is based on a semiclassical approach, which is valid for $k_{F\uparrow(\downarrow)}D \gg 1$, where $k_{F\uparrow}$ and $k_{F\downarrow}$ are the Fermi wavevectors corresponding to the two spin channels, and D is a characteristic length of the magnetization variation (DW width) [18]. In such a case DW can lead to redistribution of single-electron quasi-particles, and this can lead either to positive or negative contributions to resistivity. Another mechanism which leads to a negative contribution is based on the suppression of weak localization (WL) corrections to conductivity by DWs [10]. At sufficiently low temperatures quantum interference effects in a magnetically uniform system (without DWs) lead to an increase in the resistivity due to enhanced back scattering [19, 20]. Creation of DWs destroys the interference effects and therefore diminishes the resistivity of the system.

When, however, the DW width D is of atomic size, as in some nanoconstrictions [16], the condition of semiclassical behaviour is not fulfilled. In that case, one has to use a different approach, such as for instance the one based on the scattering matrix and Landauer formalism.

In section 2 we consider the limit of a thick DW, where electronic transport through the wall is diffusive. We take into account electron–electron interactions and analyse the charge accumulated by the wall. The limit of atomic-size DWs is considered in section 3. In particular, we consider there the case of DWs in one-dimensional systems. The role of electron–electron interactions in such a case is also discussed. Concluding remarks are given in section 4.

2. Diffusive transport through a thick domain wall

2.1. Model

Assume a simplified model of a ferromagnetic metal, in which conduction electrons with a parabolic energy spectrum interact with a nonuniform magnetization that smoothly varies across a certain DW. Assume also that the electrons are scattered by defects with the corresponding scattering potential being independent of the spin orientation (in a general

case this potential can be spin dependent). When the domain wall is sufficiently thick, $D \gg l$, where l is the electron mean free path, electronic transport across the wall is diffusive.

The single-particle Hamiltonian describing conduction electrons locally exchange-coupled to the magnetization $\mathbf{M}(\mathbf{r})$ takes the form

$$H_0 = -\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - J\boldsymbol{\sigma}\cdot\mathbf{M}(\mathbf{r}) \tag{1}$$

where J is the exchange parameter, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, and the unit system with $\hbar = 1$ is used.

The domain wall is characterized by a magnetization profile $\mathbf{M}(\mathbf{r})$. For the sake of simplicity we assume $|\mathbf{M}(\mathbf{r})| = M_0 = \text{const}$. Thus, we can write

$$J\mathbf{M}(\mathbf{r}) = M\mathbf{n}(\mathbf{r}) \tag{2}$$

where $\mathbf{n}(\mathbf{r})$ is a unit vector field specific to a particular type of DW (to be defined later), and $M = JM_0$ is measured in energy units.

In order to control the charge density of the electron gas, we include the Coulomb electron–electron interaction in the mean-field approximation via the term

$$H_{\text{int}} = e\phi(z) \tag{3}$$

where e is the electron charge ($e < 0$) and the field $\phi(z)$ is the mean-field electrostatic potential in the presence of the wall, which obeys the equation

$$\frac{\mathrm{d}^2\phi(z)}{\mathrm{d}z^2} = -4\pi e(\langle\psi^\dagger\psi\rangle - n_0) \tag{4}$$

with $\langle\cdots\rangle$ denoting the ground state average, n_0 being the electron gas density in the absence of DWs, and ψ and $\psi^\dagger$ denoting the spinor field operators. The potential $\phi(z)$ has to be calculated self-consistently, which assures that the total charge accumulated at the wall vanishes, though the charge neutrality may be violated locally.

To include the spin-dependent interaction, we introduce the contact coupling in the form

$$H^s_{\text{int}} = -\frac{g_s^2}{2}\int \mathrm{d}^3\mathbf{r}(\psi^\dagger\sigma_z\psi - s_0)^2 \tag{5}$$

where g_s is the corresponding coupling constant. Choosing s_0 as the spin density far from the wall guarantees that this interaction vanishes when there is no domain wall. This means that the effects due to magnetization of the conduction electrons in a system without domain wall are included by the parameter M in the one-particle Hamiltonian. The effect of a domain wall is then to modify the internal magnetization, resulting from a redistribution of the spin density. The effects due to interaction (5) can be taken into account by adding to the Hamiltonian a new term,

$$H^s_{\text{int}} = \int \mathrm{d}^3\mathbf{r}\, m_z(z)\psi^\dagger\sigma_z\psi \tag{6}$$

where the internal magnetization field $m_z(z)$ is determined by the saddle-point equation

$$m_z(z) = -g_s^2(\langle\psi^\dagger\sigma_z\psi\rangle - s_0). \tag{7}$$

The total Hamiltonian H of the system can then be written as

$$H = H_0 + H_{\text{int}} + H^s_{\text{int}} \tag{8}$$

where H_0, H_{int} and H^s_{int} are given by equations (1), (3) and (6), respectively.

2.2. Gauge transformation

The key point of the approach is a local unitary transformation

$$\psi \to T(\mathbf{r})\psi \qquad T^{\dagger}(\mathbf{r})T(\mathbf{r}) = \check{1} \tag{9}$$

where $\check{1}$ is the 2×2 unit matrix. $T(\mathbf{r})$ transforms the problem of electrons in a system with nonuniform magnetization to an equivalent problem of electrons in a system with uniform magnetization, but with an additional gauge field [10, 18]. In other words, $T(\mathbf{r})$ transforms the second term in equation (1) as

$$T^{\dagger}(\mathbf{r})\boldsymbol{\sigma} \cdot \mathbf{n}(\mathbf{r})T(\mathbf{r}) = \sigma_z. \tag{10}$$

Explicit form of $T(\mathbf{r})$ is given by [21]

$$T(\mathbf{r}) = \frac{1}{\sqrt{2}}\left(\check{1}\sqrt{1 + n_z(\mathbf{r})} + \mathrm{i}\frac{n_y(\mathbf{r})\sigma_x - n_x(\mathbf{r})\sigma_y}{\sqrt{1 + n_z(\mathbf{r})}}\right). \tag{11}$$

Generally, the above transformation can be applied not only to simple DWs, but also to other types of topological excitations in ferromagnetic systems, for instance to helicoidal waves, skyrmions and others.

The non-Abelian gauge field $\mathbf{A}(\mathbf{r})$ related to the transformation (11) is given by

$$\mathbf{A}(\mathbf{r}) = T^{\dagger}(\mathbf{r})\frac{\partial}{\partial \mathbf{r}}T(\mathbf{r}). \tag{12}$$

According to (11) and (13), the gauge field $\mathbf{A}(\mathbf{r})$ is a matrix in the spin space.

Assume now a more specific DW in a bulk system, which is translationally invariant in the x–y plane: $\mathbf{M}(\mathbf{r}) \to \mathbf{M}(z)$ and $\mathbf{n}(\mathbf{r}) \to \mathbf{n}(z)$. For a simple DW with $\mathbf{M}(z)$ lying in the plane normal to the wall one can parametrize the vector $\mathbf{n}(z)$ as

$$\mathbf{n}(z) = (\sin\varphi(z), 0, \cos\varphi(z)) \tag{13}$$

where the phase $\varphi(z)$ determines the type of DWs. The transformation (11) is then reduced to

$$T(z) = \frac{1}{\sqrt{2}}\left(\check{1}\sqrt{1 + \cos\varphi(z)} - \mathrm{i}\sigma_y\frac{\sin\varphi(z)}{\sqrt{1 + \cos\varphi(z)}}\right) \tag{14}$$

and the gauge field acquires the simple form

$$\mathbf{A}(z) = \left(0, 0, -\frac{\mathrm{i}}{2}\sigma_y\varphi'(z)\right) \tag{15}$$

where $\varphi'(z) \equiv \partial\varphi(z)/\partial z$.

Taking into account the above formulae one can write the full transformed Hamiltonian in the form

$$H = -\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - (M - m_z)\sigma_z + e\phi(z) + \frac{m\beta^2(z)}{2} + \mathrm{i}\sigma_y\frac{\beta'(z)}{2} + \mathrm{i}\sigma_y\beta(z)\frac{\partial}{\partial z} \tag{16}$$

where

$$\beta(z) = \frac{\varphi'(z)}{2m}. \tag{17}$$

When $k_{F\uparrow(\downarrow)}D \gg 1$, the perturbation due to the DW is weak and the semiclassical approximation is well justified. The last three terms on the right-hand side of equation (16) can then be treated as a small perturbation.

If one assumes the domain wall in the form of a kink shown schematically in figure 1, then

$$\varphi(z) = -\frac{\pi}{2}\tanh(z/L) \tag{18}$$

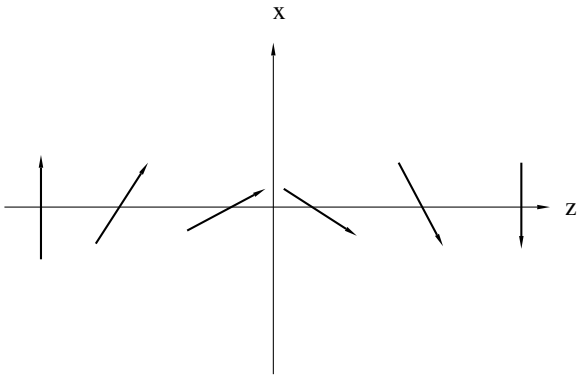

Figure 1. Variation of the magnetization in the domain wall.

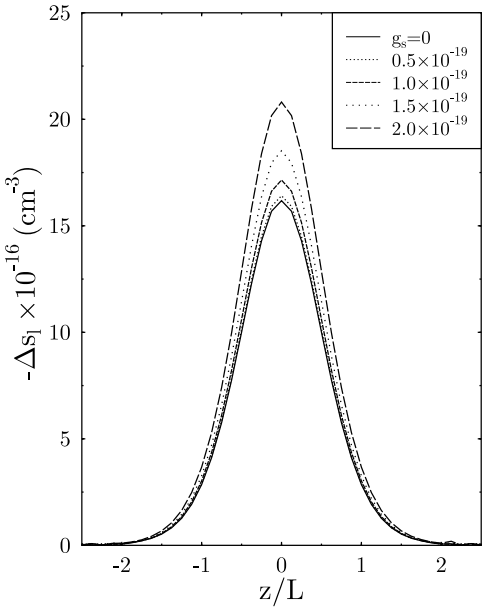

Figure 2. Distribution of the excess spin density near the domain wall for different values of the coupling constant g_s.

with $L = D/2$, and the parameter $\beta(z)$ is given by

$$\beta(z) = -\frac{\pi}{4mL\cosh^2(z/L)}. \tag{19}$$

Using equations (16) to (19), one can calculate self-consistently the distributions of spin and charge densities near the domain wall. The results are presented in figures 2 and 3. They also show how the spin coupling constant g_s affects both the spin accumulation (figure 2) and charge accumulation $\Delta\rho(z)$ (figure 3). This effect is a result of self-consistency, because by controlling the magnetic density one modifies the magnetic wall, and this in turn influences the electron density.

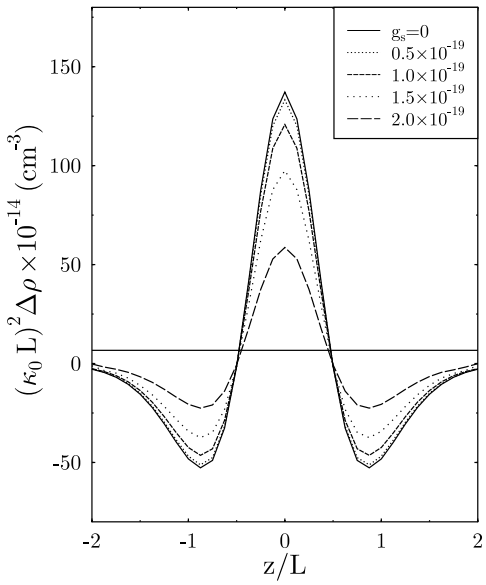

Figure 3. Distribution of the charge density near the domain wall for different values of the coupling constant g_s.

2.3. Local conductivity

The general formula for the local conductivity (without localization corrections and for electric field applied along the axis z) has the following form:

$$\sigma_{zz} = \frac{e^2}{2\pi m^2}\mathrm{Tr}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}(k_z - m\beta\sigma_y)G^R_{\mathbf{k}}\,(k_z - m\beta\sigma_y)G^A_{\mathbf{k}} \tag{20}$$

where the gauge potential $\mathbf{A}(z)$ given by equation (16) is taken into account, and the retarded (R) and advanced (A) Green functions are both evaluated at the Fermi level,

$$G^{R,A}_{\mathbf{k}} = \frac{-\varepsilon_{\mathbf{k}} - M_r\sigma_z - k_z\beta(z)\sigma_y + \mu_r(z)}{[-\varepsilon_{\mathbf{k}\uparrow}(z) + \mu_r(z) \pm \mathrm{i}/2\tau_\uparrow(z)][-\varepsilon_{\mathbf{k}\downarrow}(z) + \mu_r(z) \pm \mathrm{i}/2\tau_\downarrow(z)]}. \tag{21}$$

Here, $M_r = M - m_z$, $\varepsilon_{\mathbf{k}} = \left(q^2 + k_z^2\right)\big/2m$ with $q^2 = k_x^2 + k_y^2$, $\mu_r(z) = \mu - m\beta^2(z)/2 - e\phi(z)$ with μ denoting the chemical potential, and

$$\varepsilon_{\mathbf{k}\uparrow(\downarrow)}(z) = \varepsilon_{\mathbf{k}} \mp \left[M_r^2 + k_z^2\beta^2(z)\right]^{1/2} \tag{22}$$

where the upper (lower) sign refers to $\uparrow$ ($\downarrow$). The quasi-particle energies $\varepsilon_{\mathbf{k}\uparrow(\downarrow)}(z)$ are the eigenvalues of the whole Hamiltonian (poles of the Green functions). They correspond to pure spin states only outside the wall, whereas inside the wall they have no pure spin-up (spin-down) form because of spin mixing by the wall. Finally, $\tau_\uparrow(z)$ and $\tau_\downarrow(z)$ in equation (21) are the relaxation times, which for impurity scattering potential V_0 independent of the electron spin have the form

$$\frac{1}{\tau_{\uparrow(\downarrow)}(z)} = \frac{mV_0^2}{2\pi}\left[k_{F\uparrow}(z) + k_{F\downarrow}(z) \pm \frac{M_r}{\beta(z)}\mathrm{arcsinh}\frac{k_{F\uparrow}(z)\beta(z)}{M_r} \mp \frac{M_r}{\beta(z)}\mathrm{arcsinh}\frac{k_{F\downarrow}(z)\beta(z)}{M_r}\right] \tag{23}$$

where $k_{F\uparrow(\downarrow)}(z)$ are the appropriate Fermi wavevectors,

$$k^2_{F\uparrow(\downarrow)}(z) = 2m\mu_r(z) + 2m^2\beta^2(z) \pm 2m\left[2m\mu_r(z)\beta^2(z) + m^2\beta^4(z) + M_r^2\right]^{1/2}. \tag{24}$$

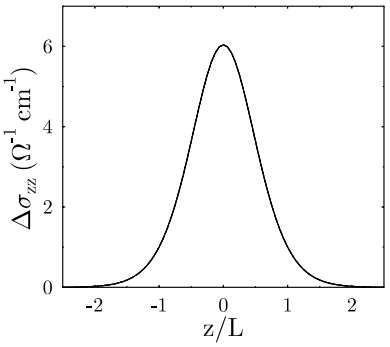

Figure 4. Domain wall contribution to local conductivity, calculated for $L = 50$ Å, Fermi energies $E_{F\uparrow} = 3$ eV and $E_{F\downarrow} = 2.5$ eV, and for impurity scattering potential leading to the bulk conductivity (without domain wall) $\sigma = 0.67 \times 10^5\ \Omega^{-1}\ \mathrm{cm}^{-1}$.

The difference in scattering times is due to a difference in the density of states at the Fermi level for $\uparrow$ and $\downarrow$ states.

The local conductivity σ_{zz} is a smoothly varying function of z, $\sigma_{zz} = \sigma_{zz}(z)$,

$$\sigma_{zz}(z) = \frac{e^2}{2\pi^2 m} \sum_{\sigma=\uparrow,\downarrow} \tau_\sigma(z) \left(\frac{k_{F\sigma}^3(z)}{3} + m^2\beta^2(z) k_{F\sigma}(z) - m^2 M_r \beta(z) \arctan \frac{k_{F\sigma}(z)\beta(z)}{M_r} \right). \tag{25}$$

Given the conductivity $\sigma_{zz}(z)$, the average resistivity of a sample of length d with a domain wall can be found as

$$\rho = \frac{1}{d} \int \frac{\mathrm{d}z}{\sigma_{zz}(z)}. \tag{26}$$

The DW contribution to the local conductivity, $\Delta\sigma_{zz}$, is shown in figure 4 as a function of z. It is evident that this contribution is positive, i.e., the conductivity is enhanced within the wall. The enhancement shown in figure 4 is not large, but it could be larger when one assumed appropriate spin asymmetry of the impurity scattering potential. It should be noted, however, that by taking opposite spin asymmetry for the impurity scattering potential, the enhancement can be diminished or can even change sign, i.e., the conductivity within the wall can be lower than outside the wall. Thus, this model can account for both signs of the magnetoresistance associated with DWs. This sign depends on the spin asymmetry of the impurity scattering potential.

3. Transport through an atomic-size domain wall

3.1. Scattering states

Let us consider again the Hamiltonian (1) describing electrons in a spatially inhomogeneous magnetization $\mathbf{M}(\mathbf{r})$. For a very narrow constrained DW one may consider only a few channels for electronic transport. A limiting situation is when there is only a single transport channel. In such a one-dimensional case the Hamiltonian (1) can be rewritten as

$$H = -\frac{1}{2m} \frac{\mathrm{d}^2}{\mathrm{d}z^2} - J M_z(z)\sigma_z - J M_x(z)\sigma_x. \tag{27}$$

We will make use of the scattering states taken in the form

$$\chi_{R\uparrow k}(z) = \begin{cases} \begin{pmatrix} \mathrm{e}^{\mathrm{i}k_\uparrow z} + r_{R\uparrow}\,\mathrm{e}^{-\mathrm{i}k_\uparrow z} \\ r^f_{R\uparrow}\,\mathrm{e}^{-\mathrm{i}k_\downarrow z} \end{pmatrix} & z \ll -L \\ \begin{pmatrix} t_{R\uparrow}\,\mathrm{e}^{\mathrm{i}k_\downarrow z} \\ t^f_{R\uparrow}\,\mathrm{e}^{\mathrm{i}k_\uparrow z} \end{pmatrix} & z \gg L \end{cases} \tag{28}$$

where $k_{\uparrow(\downarrow)} = \sqrt{2m(E \pm M)}$, and E is the electron energy. This state describes the spin-up electron wave incident from $-\infty$ and partly reflected and transmitted into the spin-up and spin-down channels. The coefficients $t_{R\uparrow}$ and $t^f_{R\uparrow}$ are the transmission amplitudes without and with spin reversal, respectively, whereas $r_{R\uparrow}$ and $r^f_{R\uparrow}$ are the relevant reflection amplitudes. The analogous forms have the scattering states related to the spin-down wave incident from left to right (labelled with $R \downarrow k$), as well as the scattering states related to electron waves incident on DW from the right.

Now we integrate the Schrödinger equation $H\psi = E\psi$ with the Hamiltonian (27) over z from $-\delta$ to $+\delta$ in the vicinity of $z = 0$ (where the domain wall is located). Assuming $L \ll \delta \ll k^{-1}_{\uparrow(\downarrow)}$, under the integral we can expand the scattering functions (28) in the vicinity of $z = 0$ and restrict ourselves to the first term in this expansion. Then one obtains

$$-\frac{1}{2m}\left(\left.\frac{\mathrm{d}\chi_n}{\mathrm{d}z}\right|_{+\delta} - \left.\frac{\mathrm{d}\chi_n}{\mathrm{d}z}\right|_{-\delta} \right) - \lambda\sigma_x\chi_n(0) = 0 \tag{29}$$

where n is the electron state index ($n \equiv R\,(L) \uparrow (\downarrow)k$) and λ is a factor defined as

$$\lambda \simeq \int_{-\infty}^{\infty} \mathrm{d}z\, J M_x(z) \simeq ML. \tag{30}$$

Equation (29) has the form of a spin-dependent condition for transmission through a δ-like potential barrier located at $z = 0$.

Taking into account the scattering states (28) and condition (29), in combination with the continuity condition for the wavefunctions, one finds the following expressions for the transmission amplitudes:

$$t_{R\uparrow(\downarrow)} = t_{L\downarrow(\uparrow)} = \frac{2v_{\uparrow(\downarrow)}(v_\uparrow + v_\downarrow)}{(v_\uparrow + v_\downarrow)^2 + 4\lambda^2} \tag{31}$$

$$t^f_{R\uparrow(\downarrow)} = t^f_{L\downarrow(\uparrow)} = \frac{4\mathrm{i}\lambda v_{\uparrow(\downarrow)}}{(v_\uparrow + v_\downarrow)^2 + 4\lambda^2} \tag{32}$$

where $v_{\uparrow(\downarrow)} = k_{\uparrow(\downarrow)}/m$.

According to (32), the magnitude of the spin-flip transmission coefficient can be estimated as

$$|t^f|^2 \sim \left(\frac{\lambda v}{v^2 + \lambda^2}\right)^2 \sim \left(\frac{M\varepsilon_0}{\varepsilon_F\varepsilon_0 + M^2}\right)^2 (k_F L)^2 \tag{33}$$

where $\varepsilon_F = k_F^2/2m$, and $\varepsilon_0 = 1/mL^2$. For $k_F L \ll 1$ one finds $\varepsilon_0 \gg \varepsilon_F$. Taking $\varepsilon_F \sim M$, one obtains

$$|t^f|^2 \sim \left(\frac{M}{\varepsilon_F} k_F L\right)^2 \ll 1. \tag{34}$$

Thus, a sharp domain wall can be considered as an effective barrier for the spin-flip transmission.

It should be noted that the conservation of flow has the following form:

$$v_\uparrow(1-|r_{R\uparrow}|^2)-v_\downarrow\left|r^f_{R\uparrow}\right|^2=v_\downarrow|t_{R\uparrow}|^2+v_\uparrow\left|t^f_{R\uparrow}\right|^2 \tag{35}$$

and analogous equations hold also for the other scattering states.

3.2. Resistance of the domain wall

To calculate the conductivity we start from the current operator

$$\hat{j}(z)=e\psi^\dagger(z)\hat{v}\psi(z). \tag{36}$$

Expanding $\psi(z)$ in the scattering states (28), and performing quantum mechanical averaging, one obtains the following formula for the current:

$$j(z)=-\mathrm{i}e\sum_n\int\frac{\mathrm{d}\varepsilon}{2\pi}\,\mathrm{e}^{\mathrm{i}\varepsilon\delta}G_n(\varepsilon)\chi^\dagger_n(z)\hat{v}\chi_n(z) \tag{37}$$

where n is the index of scattering states. The matrix elements of the velocity operator $\hat{v}=-(\mathrm{i}/m)\partial_z$ can be calculated on the basis of the scattering states, and one obtains

$$v_{R\uparrow(\downarrow)}\equiv\langle R\uparrow(\downarrow)k|\hat{v}|R\uparrow(\downarrow)k\rangle=v_{\downarrow(\uparrow)}|t_{R\uparrow(\downarrow)}|^2+v_{\uparrow(\downarrow)}\left|t^f_{R\uparrow(\downarrow)}\right|^2 \tag{38}$$

and similar expressions for the other states.

The retarded Green function $G_n(\varepsilon)$ in equation (37) is diagonal on the basis of scattering states. Assuming that the transmission of electrons through the barrier is small, one can take the chemical potential constant $\mu=\mu_R$ for $z<0$, and $\mu=\mu_L$ for $z>0$. This corresponds to the voltage drop $U=(\mu_R-\mu_L)/e$ across the barrier. The Green function $G_{R\uparrow k}(\varepsilon)$ acquires then the following simple form:

$$G_{R\uparrow k}(\varepsilon)=\frac{1}{\varepsilon-\varepsilon_{R\uparrow}(k)+\mu_R+\mathrm{i}\delta} \tag{39}$$

where $\varepsilon_{R\uparrow}(k)=k^2/2m-M$. The other components of the Green function have a similar form.

After integrating over ε, one finds

$$\begin{aligned}j(z)=e\int\frac{\mathrm{d}k}{2\pi}\Big\{&v_\uparrow\chi^\dagger_{R\uparrow k}(z)\chi_{R\uparrow k}(z)\theta[\mu_R-\varepsilon_{R\uparrow}(k)]+v_\downarrow\chi^\dagger_{R\downarrow k}(z)\chi_{R\downarrow k}(z)\theta[\mu_R-\varepsilon_{R\downarrow}(k)]\\&-v_\uparrow\chi^\dagger_{L\uparrow k}(z)\chi_{L\uparrow k}(z)\theta[\mu_L-\varepsilon_{L\uparrow}(k)]-v_\downarrow\chi^\dagger_{L\downarrow k}(z)\chi_{L\downarrow k}(z)\theta[\mu_L-\varepsilon_{L\downarrow}(k)]\Big\}.\end{aligned} \tag{40}$$

In view of the conservation of charge, the current does not depend on z, and therefore can be calculated for $z=0$. Moreover, the total current from the states $\varepsilon_{R\uparrow(\downarrow)}(k),\varepsilon_{L\uparrow(\downarrow)}(k)\leqslant\mu_L$ vanishes and only the states obeying the condition $\mu_L<\varepsilon_{R\uparrow(\downarrow)}(k)<\mu_R$ contribute to the current. The conductance G can then be found as a linear response to small perturbations (in the limit of $U\to 0$), and one finds

$$G=\frac{e^2}{2\pi}\left(\frac{v_\downarrow}{v_\uparrow}|t_{R\uparrow}|^2+\left|t^f_{R\uparrow}\right|^2+\frac{v_\uparrow}{v_\downarrow}|t_{R\downarrow}|^2+\left|t^f_{R\downarrow}\right|^2\right) \tag{41}$$

where all the velocities and transmission coefficients are taken at the Fermi level.

Finally, using equations (31) and (32), one can write the conductance in the form

$$G=\frac{4e^2}{\pi}\frac{v_\uparrow v_\downarrow(v_\uparrow+v_\downarrow)^2+2\lambda^2\left(v_\uparrow^2+v_\downarrow^2\right)}{[(v_\uparrow+v_\downarrow)^2+4\lambda^2]^2}. \tag{42}$$

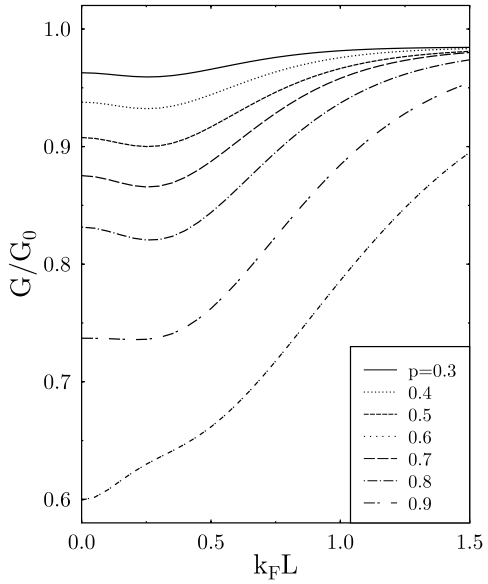

Figure 5. Relative conductance of the magnetic quantum wire in the presence of a domain wall as a function of the domain wall width L. The different curves are calculated for indicated values of the parameter $p = M/\varepsilon_F$.

In the limit of $v_\uparrow = v_\downarrow$ and $\lambda \to 0$ one obtains the conductance of a single spin-degenerate channel, $G_0 = e^2/\pi$.

The dependence of G/G_0 on the wall parameter L is shown in figure 5 for different values of the parameter $p = M/\varepsilon_F$. We calculated this dependence numerically, whereas solution (42) describes the dependence on L at small $L \ll k_F^{-1}$. Thus, the results shown in figure 5 are valid for arbitrary values of $k_F L$. The numerical modelling has been done by direct calculation of the spinor wavefunction using equation (27), starting at $z \gg L$ in the form of two transmitted spin-up and spin-down waves with arbitrary numerical coefficients. Then we restored the function in the region $z \ll -L$ and, by numerically projecting the obtained spin components on the right- and left-moving waves (in accordance with equation (28)), we found the amplitudes of incident and reflected waves.

One can note that the conductance in the presence of a domain wall is generally much smaller than in the absence of the wall. Accordingly, the associated magnetoresistance can be rather large (about 70%, which corresponds to $G/G_0 < 0.6$), in agreement with experimental observations. It is also worth noting that the resistance of an abrupt domain wall can be smaller than the resistance of a thicker domain wall (provided the conditions assumed for the model are fulfilled).

3.3. Electron–electron interaction

Electron–electron interactions in one-dimensional systems can play a crucial role in electronic conductivity. This is because electron tunnelling even through a weak impurity potential can be renormalized to suppress completely the transmission coefficients. Here, we calculate the electron–electron corrections to the transmission coefficients through a domain wall, assuming that transmission with and without spin reversal is small.

Let us consider the model with the point-like electron–electron interaction described by the following term in the Hamiltonian

$$H_{\rm int} = \frac{g}{2}(\psi^\dagger\psi)^2 + \frac{g_s}{2}(\psi^\dagger\sigma_z\psi)^2 \tag{43}$$

where g and g_s correspond to the charge–charge interaction and the spin-dependent exchange interaction, respectively.

Following the method of Matveev *et al* [25] one finds the first-order correction to the scattering states as

$$\psi(z) = \psi_0(z) + \int_{-\infty}^{\infty} {\rm d}z' G_0(z, z')\Sigma(z')\psi_0(z') \tag{44}$$

where $\Sigma(z)$ is the self-energy. Restricting considerations to the point-like interaction, we calculate the self-energy as a Hartree diagram which includes two contributions corresponding to the two interaction terms. The Green function $G_0(z, z')$ in equation (44) is a matrix in the space of scattering states. As the basis functions we use the scattering states calculated in the case of no electron–electron interaction.

Using equation (44), one can calculate corrections to the transmission coefficients t and t^f in the first order of small transmission. After calculating the renormalized amplitudes [25], one finds the following dependence of the transmission coefficients near the Fermi level

$$t_{R\uparrow} \simeq t^{(0)}_{R\uparrow}\left(\frac{k - k_F}{k_F}\right)^{\gamma} \tag{45}$$

$$t^f_{R\uparrow} \simeq t^{f(0)}_{R\uparrow}\left(\frac{k - k_F}{k_F}\right)^{\gamma_f} \tag{46}$$

where

$$\gamma = \frac{r_{L\uparrow}}{4\pi v_\downarrow}\left[\frac{g + g_s}{2}(r_{R\uparrow} + r_{L\uparrow}) + (g - g_s)r_{L\uparrow})\right] \tag{47}$$

$$\gamma_f = \frac{r_{L\downarrow}}{4\pi v_\downarrow}\frac{g + g_s}{2}. \tag{48}$$

Thus, the electron–electron interaction suppresses to zero electron transmission near the Fermi level. As a result, the low-field conductance in 1D systems drops to zero in the presence of a domain wall.

4. Conclusions

We have presented a theoretical description of the domain wall contribution to the electrical resistivity of metallic ferromagnets. Two limiting cases were analysed in detail—the case of a thick domain wall with diffusive electron transport across the wall, and the limit of an atomic-size and constrained domain wall, which effectively could be described by a one-dimensional model. These two possibilities are not the only ones. In very pure systems electronic transport across a thick domain wall can be ballistic, despite the fact that the domain wall itself may be considered quasi-classically [24]. Apart from this, transport in real nanoconstrictions involves more channels and should be described by a more general theory. However, such an approach may be useful particularly in the cases of point contacts based on new semiconductors heavily doped with magnetic impurities, such as ferromagnetic GaMnAs or related compounds.

In the case of a smooth domain wall, we found a small positive contribution to the conductivity near the wall. However, our calculations show that for a choice of realistic parameters, this contribution is of the order of 0.01%. In the opposite case of a sharp domain wall, the theory gives a large negative magnetoresistance.

Acknowledgments

This work is supported by the Polish Committee for Scientific Research through the grants 5 P03B 091 20 and PBZ/KBN/044/P03/2001.

References

[1] Cabrera G G and Falicov L M 1974 *Phys. Status Solidi* b **61** 539
Cabrera G G and Falicov L M 1974 *Phys. Status Solidi* b **62** 217
[2] Berger L 1978 *J. Appl. Phys.* **49** 2156
[3] Hong K and Giordano N 1998 *J. Phys: Condens. Matter* **13** L401
[4] Rüdiger U, Yu J, Zhang S, Kent A D and Parkin S S P 1998 *Phys. Rev. Lett.* **80** 5639
[5] Kent A D, Rüdiger U, Yu J, Thomas L and Parkin S S P 1999 *J. Appl. Phys.* **85** 5243
[6] Kent A D, Yu J, Rüdiger U and Parkin S S P 2001 *J. Phys.: Condens. Matter* **13** R461
[7] Gregg J F, Allen W, Ounadjela K, Viret M, Hehn M, Thompson S M and Coey J M D 1996 *Phys. Rev. Lett.* **77** 1580
[8] Garcia N, Muñoz M and Zhao Y W 1999 *Phys. Rev. Lett.* **82** 2923
[9] Ebels U, Radulescu A, Henry Y, Piraux L and Ounadjela K 2000 *Phys. Rev. Lett.* **84** 983
[10] Tatara G and Fukuyama H 1997 *Phys. Rev. Lett.* **78** 3773
[11] Levy P M and Zhang S 1997 *Phys. Rev. Lett.* **79** 5110
[12] van Gorkom R P, Brataas A and Bauer G E W 1999 *Phys. Rev. Lett.* **83** 4401
[13] Jonkers P A E, Pickering S J, De Raedt H and Tatara G 1999 *Phys. Rev.* B **60** 15970
[14] Tatara G 2001 *Int. J. Mod. Phys.* B **15** 321
[15] Chopra H D and Hua S Z 2002 *Phys. Rev.* B **66** 020403(R)
[16] Bruno P 1999 *Phys. Rev. Lett.* **83** 2425
[17] Imamura H, Kobayashi N, Takahashi S and Maekawa S 2000 *Phys. Rev. Lett.* **84** 1003
[18] Dugaev V K, Barnaś J, Łusakowski A and Turski L A 2002 *Phys. Rev.* B **65** 224419
[19] Altshuler B L, Aronov A G, Khmelnitskii D E and Larkin A I 1982 *Quantum Theory of Solids* (Moscow: Mir) p 130
[20] Lee P A and Ramakrishnan T V 1985 *Rev. Mod. Phys.* **57** 287
[21] Falko V I and Iordanskii S V 1999 *Phys. Rev. Lett.* **82** 402
[22] Lyanda-Geller Y, Aleiner I L and Goldbart P M 1998 *Phys. Rev. Lett.* **81** 3215
[23] Dugaev V K, Bruno P and Barnaś J 2001 *Phys. Rev.* B **64** 144423
[24] Tagirov L R, Vodopyanov B P and Efetov K B 2001 *Phys. Rev.* B **63** 104428
[25] Matveev K A, Yue D and Glazman L I 1993 *Phys. Rev. Lett.* **71** 3351